4.5 Prove Triangles Congruent by ASA and AAS

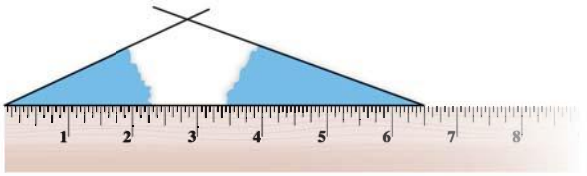
Before You used the SSS, SAS, and HL congruence methods.

Now You will use two more methods to prove congruences.

Why? So you can recognize congruent triangles in bikes, as in Exs. 23–24.

Key Vocabulary
- **flow proof**

Suppose you tear two angles out of a piece of paper and place them at a fixed distance on a ruler. Can you form more than one triangle with a given length and two given angle measures as shown below?



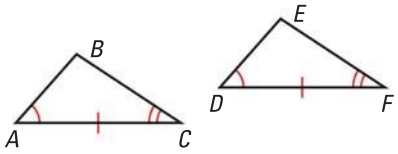
In a polygon, the side connecting the vertices of two angles is the *included* side. Given two angle measures and the length of the included side, you can make only one triangle. So, all triangles with those measurements are congruent.

THEOREMS *For Your Notebook*

POSTULATE 21 Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

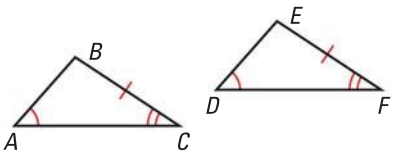
If Angle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.6 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

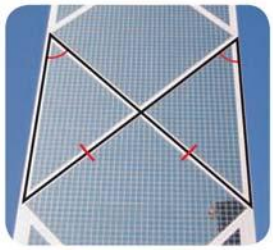
If Angle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Proof: Example 2, p. 250

EXAMPLE 1 Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. 

b. 

c. 

Solution

a. The vertical angles are congruent, so two pairs of angles and a pair of non-included sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

AVOID ERRORS
You need at least one pair of congruent corresponding sides to prove two triangles congruent.

b. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

c. Two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

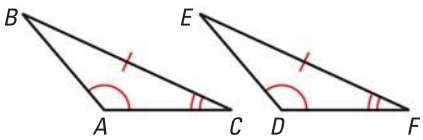
FLOW PROOFS You have written two-column proofs and paragraph proofs. A **flow proof** uses arrows to show the flow of a logical argument. Each reason is written below the statement it justifies.

EXAMPLE 2 Prove the AAS Congruence Theorem

Prove the Angle-Angle-Side Congruence Theorem.

GIVEN ▶ $\angle A \cong \angle D, \angle C \cong \angle F, \overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ — Given

$\angle B \cong \angle E$ — Third ∡ Thm.

$\angle C \cong \angle F$ — Given

$\triangle ABC \cong \triangle DEF$ — ASA Congruence Post.

$\overline{BC} \cong \overline{EF}$ — Given

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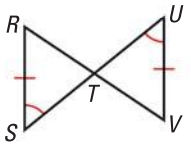
✓ GUIDED PRACTICE for Examples 1 and 2

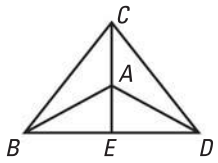
1. In the diagram at the right, what postulate or theorem can you use to prove that $\triangle RST \cong \triangle VUT$? *Explain.*



2. Rewrite the proof of the Triangle Sum Theorem on page 219 as a flow proof.

EXAMPLE 3 Write a flow proof

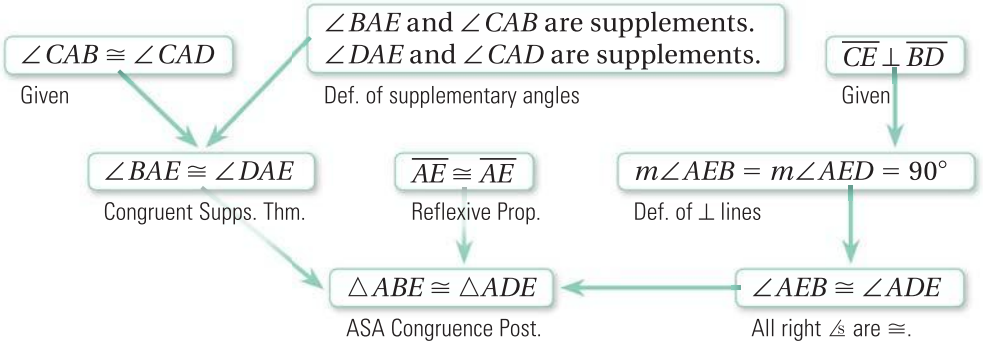
In the diagram, $\overline{CE} \perp \overline{BD}$ and $\angle CAB \cong \angle CAD$. Write a flow proof to show $\triangle ABE \cong \triangle ADE$.



Solution

GIVEN ▶ $\overline{CE} \perp \overline{BD}$, $\angle CAB \cong \angle CAD$

PROVE ▶ $\triangle ABE \cong \triangle ADE$

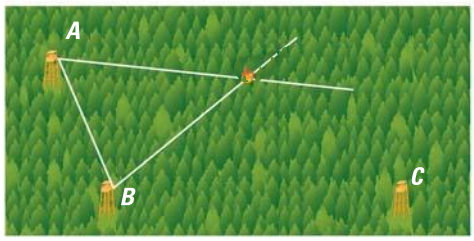


EXAMPLE 4 Standardized Test Practice

FIRE TOWERS The forestry service uses fire tower lookouts to watch for forest fires. When the lookouts spot a fire, they measure the angle of their view and radio a dispatcher. The dispatcher then uses the angles to locate the fire. How many lookouts are needed to locate a fire?

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ Not enough information

The locations of tower A, tower B, and the fire form a triangle. The dispatcher knows the distance from tower A to tower B and the measures of $\angle A$ and $\angle B$. So, he knows the measures of two angles and an included side of the triangle.



By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the fire location is given by the third vertex. Two lookouts are needed to locate the fire.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 3 and 4

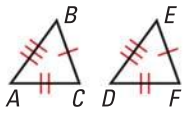
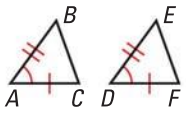
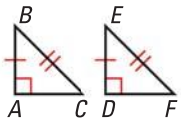
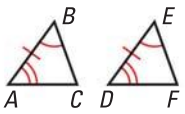
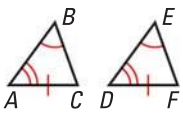
3. In Example 3, suppose $\angle ABE \cong \angle ADE$ is also given. What theorem or postulate besides ASA can you use to prove that $\triangle ABE \cong \triangle ADE$?

4. **WHAT IF?** In Example 4, suppose a fire occurs directly between tower B and tower C. Could towers B and C be used to locate the fire? *Explain*.

CONCEPT SUMMARY *For Your Notebook*

Triangle Congruence Postulates and Theorems

You have learned five methods for proving that triangles are congruent.

SSS	SAS	HL (right △ only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **A**ngle-**L**eg, **L**eg-**L**eg, and **H**ypotenuse-**A**ngle.

4.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 21, and 26

SKILL PRACTICE

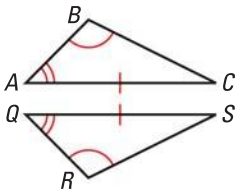
1. **VOCABULARY** Name one advantage of using a flow proof rather than a two-column proof.

2. ★ **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

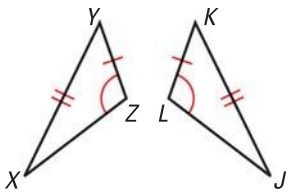
EXAMPLE 1 on p. 250 for Exs. 3–7

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.**

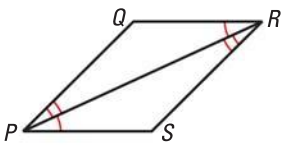
3. $\triangle ABC$, $\triangle QRS$



4. $\triangle XYZ$, $\triangle JKL$

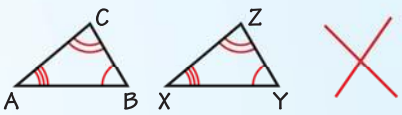


5. $\triangle PQR$, $\triangle RSP$

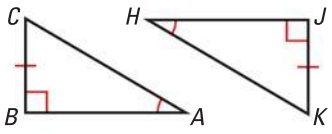


6. **ERROR ANALYSIS** *Describe* the error in concluding that $\triangle ABC \cong \triangle XYZ$.

By AAA, △ABC ≅ △XYZ.



7. ★ **MULTIPLE CHOICE** Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle HJK$?



Ⓐ ASA Ⓑ AAS

Ⓒ SAS Ⓓ Not enough information

EXAMPLE 2 on p. 250 for Exs. 8–13

DEVELOPING PROOF **State the third congruence that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given postulate or theorem.**

8. **GIVEN** ▶ $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, ? $\cong$?

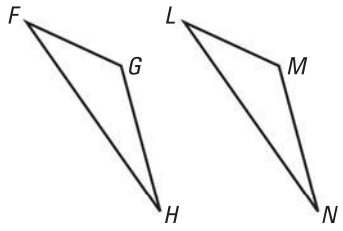
Use the AAS Congruence Theorem.

9. **GIVEN** ▶ $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, ? $\cong$?

Use the ASA Congruence Postulate.

10. **GIVEN** ▶ $\overline{FH} \cong \overline{LN}$, $\angle H \cong \angle N$, ? $\cong$?

Use the SAS Congruence Postulate.

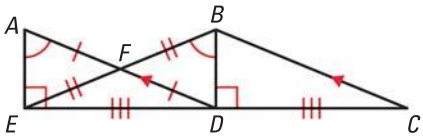


OVERLAPPING TRIANGLES ***Explain*** **how you can prove that the indicated triangles are congruent using the given postulate or theorem.**

11. $\triangle AFE \cong \triangle DFB$ by SAS

12. $\triangle AED \cong \triangle BDE$ by AAS

13. $\triangle AED \cong \triangle BDC$ by ASA



DETERMINING CONGRUENCE **Tell whether you can use the given information to determine whether $\triangle ABC \cong \triangle DEF$. *Explain* your reasoning.**

14. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

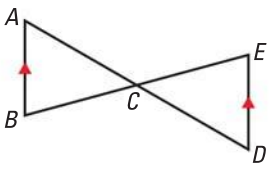
15. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

16. $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DE}$

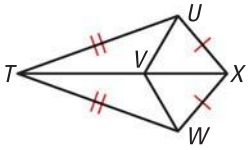
17. $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FD}$, $\overline{AC} \cong \overline{DE}$

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate(s) or theorem(s) you would use.**

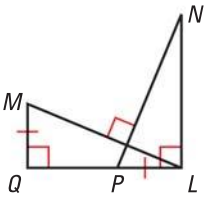
18. $\triangle ABC$, $\triangle DEC$



19. $\triangle TUV$, $\triangle TWV$



20. $\triangle QML$, $\triangle LPN$

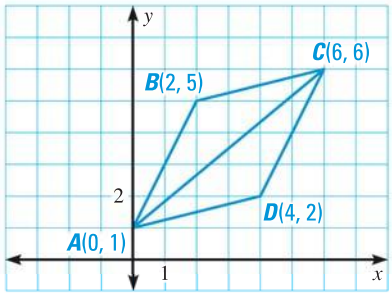


21. ★ **EXTENDED RESPONSE** Use the graph at the right.

a. Show that $\angle CAD \cong \angle ACB$. *Explain* your reasoning.

b. Show that $\angle ACD \cong \angle CAB$. *Explain* your reasoning.

c. Show that $\triangle ABC \cong \triangle CDA$. *Explain* your reasoning.



22. **CHALLENGE** Use a coordinate plane.

a. Graph the lines $y = 2x + 5$, $y = 2x - 3$, and $x = 0$ in the same coordinate plane.

b. Consider the equation $y = mx + 1$. For what values of m will the graph of the equation form two triangles if added to your graph? For what values of m will those triangles be congruent? *Explain*.

PROBLEM SOLVING

CONGRUENCE IN BICYCLES *Explain* **why the triangles are congruent.**

23.

24.

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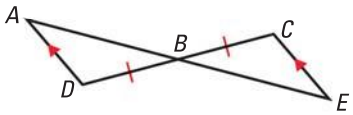
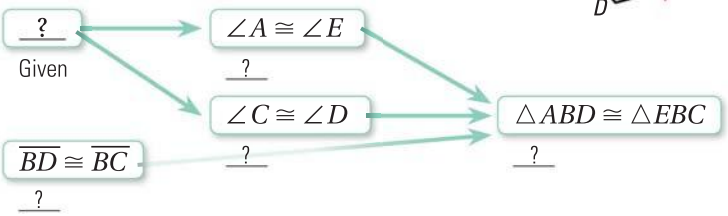
EXAMPLE 3 on p. 251 for Ex. 25

25. FLOW PROOF Copy and complete the flow proof.

GIVEN ▶ $\overline{AD} \parallel \overline{CE}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$





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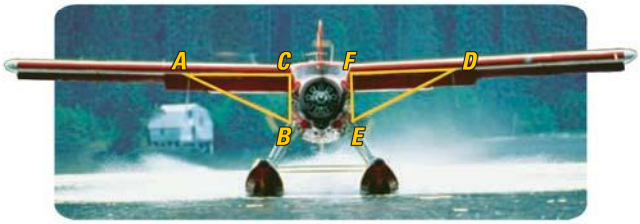
EXAMPLE 4 on p. 251 for Ex. 26

26. ★ SHORT RESPONSE You are making a map for an orienteering race. Participants start at a large oak tree, find a boulder 250 yards due east of the oak tree, and then find a maple tree that is 50° west of north of the boulder and 35° east of north of the oak tree. Sketch a map. Can you locate the maple tree? *Explain*.

27. AIRPLANE In the airplane at the right, $\angle C$ and $\angle F$ are right angles, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$. What postulate or theorem allows you to conclude that $\triangle ABC \cong \triangle DEF$?



RIGHT TRIANGLES **In Lesson 4.4, you learned the Hypotenuse-Leg Theorem for right triangles. In Exercises 28–30, write a paragraph proof for these other theorems about right triangles.**

28. Leg-Leg (LL) Theorem If the legs of two right triangles are congruent, then the triangles are congruent.

29. Angle-Leg (AL) Theorem If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.

30. Hypotenuse-Angle (HA) Theorem If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent.

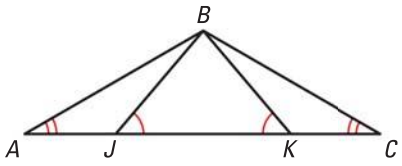
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

31. PROOF Write a two-column proof.

GIVEN ▶ $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

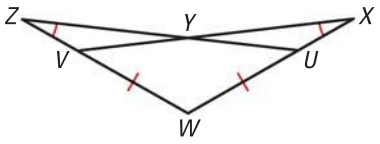
PROVE ▶ $\triangle ABK \cong \triangle CBJ$



32. PROOF Write a flow proof.

GIVEN ▶ $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

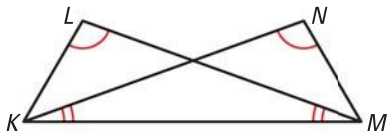
PROVE ▶ $\triangle XWV \cong \triangle ZWU$



33. PROOF Write a proof.

GIVEN ▶ $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

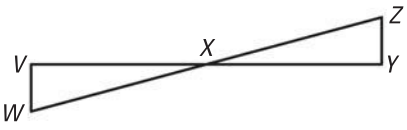
PROVE ▶ $\triangle NMK \cong \triangle LKM$



34. PROOF Write a proof.

GIVEN ▶ X is the midpoint of $\overline{VY}$ and $\overline{WZ}$.

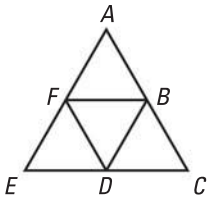
PROVE ▶ $\triangle VWX \cong \triangle YZX$



35. CHALLENGE Write a proof.

GIVEN ▶ $\triangle ABF \cong \triangle DFB$, F is the midpoint of $\overline{AE}$, B is the midpoint of $\overline{AC}$.

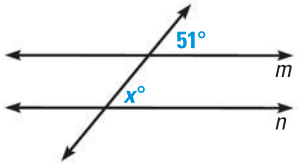
PROVE ▶ $\triangle FDE \cong \triangle BCD \cong \triangle ABF$



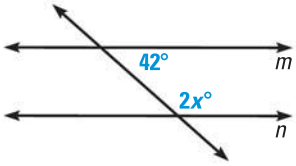
MIXED REVIEW

Find the value of x that makes $m \parallel n$. *(p. 161)*

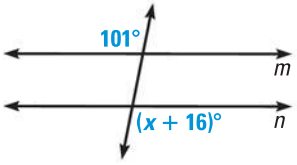
36.



37.



38.



Write an equation of the line that passes through point P and is parallel to the line with the given equation. *(p. 180)*

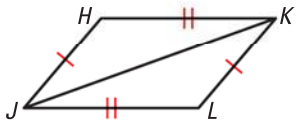
39. $P(0, 3)$, $y = x - 8$

40. $P(-2, 4)$, $y = -2x + 3$

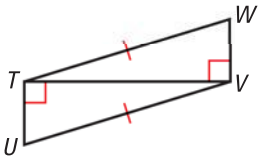
PREVIEW
Prepare for Lesson 4.6 in Exs. 41–43.

Decide which method, SSS, SAS, or HL, can be used to prove that the triangles are congruent. *(pp. 234, 240)*

41. $\triangle HJK \cong \triangle LKJ$



42. $\triangle UTV \cong \triangle WVT$



43. $\triangle XYZ \cong \triangle RQZ$